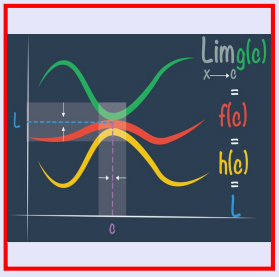
Calculus I

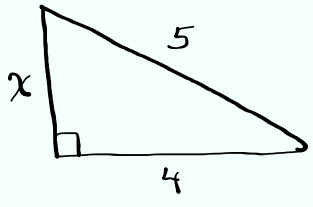
Lecture 4



Feb 19-8:47 AM

More on algebra

Find x:

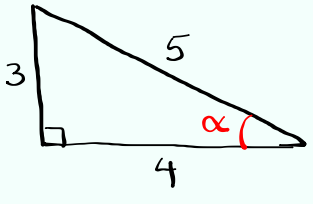


Right Triangle

Pythagorean Thrm

$x^2 + 4^2 = 5^2$

$x^2 + 16 = 25 \rightarrow \boxed{x = 3}$



$\sin\alpha = \frac{\text{opp.}}{\text{hyp.}} = \frac{3}{5}$ $\quad \csc\alpha = \frac{5}{3}$

$\cos\alpha = \frac{\text{adj.}}{\text{hyp.}} = \frac{4}{5}$ $\quad \sec\alpha = \frac{5}{4}$

$\tan\alpha = \frac{\text{opp.}}{\text{adj.}} = \frac{3}{4}$ $\quad \cot\alpha = \frac{4}{3}$

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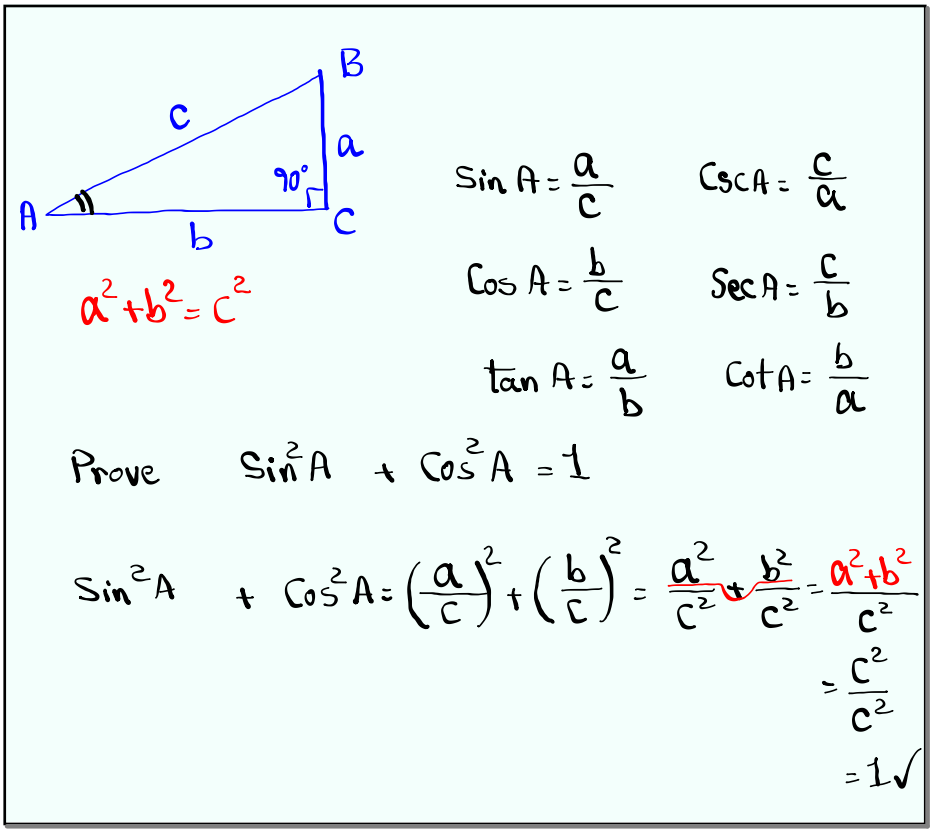

$a^2+b^2=c^2$

$\sin A=\frac{a}{c}$ $\quad \csc A=\frac{c}{a}$

$\cos A=\frac{b}{c}$ $\quad \sec A=\frac{c}{b}$

$\tan A=\frac{a}{b}$ $\quad \cot A=\frac{b}{a}$

Prove $\sin^2 A+\cos^2 A=1$

$$\sin^2 A+\cos^2 A=\left(\frac{a}{c}\right)^2+\left(\frac{b}{c}\right)^2=\frac{a^2}{c^2}+\frac{b^2}{c^2}=\frac{a^2+b^2}{c^2}$$

$$=\frac{c^2}{c^2}$$

$$=1\checkmark$$

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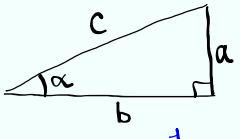
Prove $1+\tan^2\alpha=\sec^2\alpha$



$$1+\tan^2\alpha=1+\frac{a^2}{b^2}=\frac{b^2}{b^2}+\frac{a^2}{b^2}$$

$$=\frac{b^2+a^2}{b^2}=\frac{c^2}{b^2}$$

$$=\left(\frac{c}{b}\right)^2=\sec^2\alpha$$

$$\sec\alpha=\frac{1}{\cos\alpha}=\frac{1}{\frac{b}{c}}=\frac{c}{b}$$

Famous Trig. Identities:

$\sin^2\alpha+\cos^2\alpha=1$ $\quad 1+\tan^2\alpha=\sec^2\alpha$

$\tan\alpha=\frac{\sin\alpha}{\cos\alpha}$ $\quad 1+\cot^2\alpha=\csc^2\alpha$

$\cot\alpha=\frac{\cos\alpha}{\sin\alpha}$ $\quad \csc\alpha=\frac{1}{\sin\alpha}$

$\sec\alpha=\frac{1}{\cos\alpha}$

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Simplify $(\sin\alpha+\cos\alpha)^2-(\sin\alpha-\cos\alpha)^2$

Recall

$(A+B)^2=A^2+2AB+B^2$

$(A-B)^2=A^2-2AB+B^2$

$=\sin^2\alpha+2\sin\alpha\cos\alpha+\cos^2\alpha-\sin^2\alpha+2\sin\alpha\cos\alpha-\cos^2\alpha$

$=4\sin\alpha\cos\alpha$

$=2\cdot 2\sin\alpha\cos\alpha$

$=2\sin 2\alpha$

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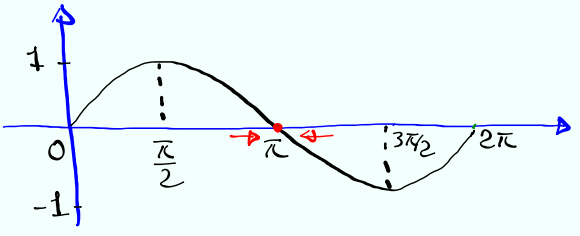
$f(x)=\sin x \qquad 0\le x\le 2\pi$

Graph



Domain: $[0,2\pi]$, Range: $[-1,1]$

As $x\to\pi^+$, $f(x)=\sin x\to 0$

As $x\to\pi^-$, $f(x)=\sin x\to 0$

$\lim_{x\to\pi^+} f(x)=0$

$\lim_{x\to\pi^-} f(x)=0$

$\lim_{x\to\pi} f(x)=0$

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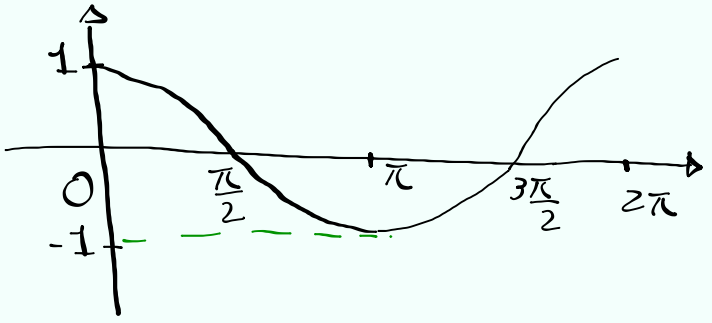
$f(x) = \cos x$ $\quad 0 \le x \le 2\pi$



$\lim_{x \to \pi^+} \cos x = -1$ $\quad \lim_{x \to \pi^-} \cos x = -1$ $\quad \lim_{x \to \pi} \cos x = -1$

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Given

$$f(x) = \begin{cases} |x| & \text{for } x < 0 \\ 1 + \sqrt{x} & \text{for } x \ge 0 \end{cases}$$

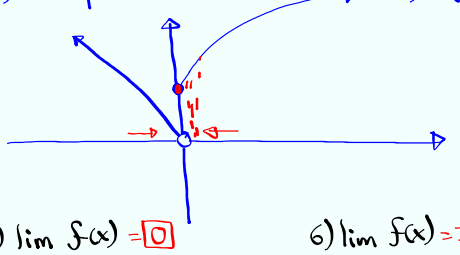
Find

1) $f(-2) = |-2| = 2$

2) $f(0) = 1 + \sqrt{0} = 1$

3) $f(4) = 1 + \sqrt{4} = 3$

4) Graph $f(x)$



5) $\lim_{x \to 0^-} f(x) = 0$

6) $\lim_{x \to 0^+} f(x) = 1$

7) $\lim_{x \to 0} f(x)$ Does not exist.

$\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

Aug 29-7:54 AM

$f(x) = |x^2 - 4|$

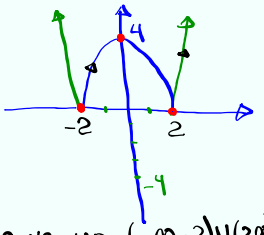
1) Y-Int $(0,4)$

2) X-Int $(\pm 2, 0)$ $f(x)=0$ $|x^2-4|=0$ $x=2,\ x=-2$

3) Domain $(-\infty,\infty)$ 4) Range $[0,\infty)$

5) Graph

Let's graph $y=x^2-4$



Increasing $(-2,0) \cup (2,\infty)$

Decreasing $(-\infty,-2) \cup (0,2)$

Concave down $(-2,2)$

Concave up $(-\infty,-2) \cup (2,\infty)$

$\lim_{x\to 0} f(x) = 4$ $\lim_{x\to 2} f(x) = 0$ $\lim_{x\to -2} f(x) = 0$

Aug 29-8:03 AM

Class QZ 1

Given $f(x) = 3x^2 - 5x - 8$

1) Y-Int $f(0) = 3(0)^2 - 5(0) - 8 = -8 \Rightarrow \boxed{(0,-8)}$

2) X-Int. $f(x)=0$ $3x^2 - 5x - 8 = 0$

$(3x - 8)(x + 1) = 0$

$x = \frac{8}{3}$ $x = -1$

$\Rightarrow \boxed{\left(\frac{8}{3},0\right),\ (-1,0)}$

Aug 29-8:18 AM